Improved Bubble Velocity Equation for Bubbling Fluidized Beds

An improved bubble velocity equation is developed for gas-solid fluidized beds. Model parameters are evaluated from reported experimental results of bubble flow. Finally, this improved equation is compared to the commonly applied velocity relationship of Davidson and Harrison (1963).

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SCOPE

Two-phase bubble models of fluidized beds require a predictive equation for the absolute upward velocity of the bubble phase. The widely used bubble velocity equation of Davidson and Harrison (1963) was derived based on the concept of a single bubble rising in an infinite medium. Hence, this equation cannot account for interference among rising bubbles reducing their velocity. In addition, there is no accounting for nonuniform flow and bubble volume fraction or for the bubble flow being less than excess flow above that at minimum fluidization. The inability to describe these effects causes significant deviations between actual bubble velocity and that predicted by the Davidson and Harrison equation.

In this work, a bubble velocity equation is derived from concepts quite different from those applied by previous investigators; yet the result is quite similar to the traditional equation. Here, a mixture model incorporating multiple-particle fluid properties is considered. Also, this analysis takes into account both effects of nonuniform flow and bubble volume fraction as well as the effect of local relative velocity between phases by inclusion of a radial distribution coefficient and weighed-mean drift velocity. The distribution coefficient is evaluated from available experimental results of local and average bubble flow properties. The form of the weighted-mean drift velocity equation is obtained by assuming churn-turbulent flow throughout the fluidized bed and by considering a multiple-particle drag coefficient for spherical-cap bubbles.

Finally, the improved bubble velocity equation is compared to the equation of Davidson and Harrison. It is demonstrated that, under the appropriate restrictive assumptions, the former reduces to the latter.

CONCLUSIONS AND SIGNIFICANCE

An improved bubble velocity equation for bubbling fluidized beds has been developed. It has the form

$$\bar{u}_B = C_o \langle u_{Bo} \rangle + 0.71 \left[\frac{g\bar{d}_b(\rho_D - \rho_B)(1 - \langle \delta_B \rangle)}{\rho_D} \right]^{1/2}$$

This equation accounts for a number of factors which contribute to deviation of actual absolute bubble velocity from that predicted by the equation of Davidson and Harrison including

- Interference between adjacent bubbles reducing bubble velocity
- Influence of nonuniform bubble distribution either to increase (for bubbles concentrated near the axial centerline) or to decrease (for bubbles closer to the wall) the absolute bubble velocity
- Reduction in bubble phase velocity which results from bubble flow being less than the excess flow above minimum fluidization

The distribution coefficient, C_o, has been evaluated from available experimental data of a number of reported investigations of bubble flow. It has been shown that calculated values agree well with qualitative aspects of its significance. In particular, bubbles are concentrated near the wall close to the inlet gas distributor and the local value of C_o is less than one. As coalescence takes place and bubbles tend toward the axial centerline, the distribution coefficient approaches 1.5. Farther up the bed, C_o can exceed 1.5 and tends to level off.

It can be concluded that this new bubble velocity equation represents a significant improvement over the widely applied equation of Davidson and Harrison. Also, since the new equation can be applied to accommodate either local (at any axial position) or bed volume-average bubble properties, it is immediately useful for interpreting experimental data and for inclusion in comprehensive models of fluidized-bed hydrodynamics.

BACKGROUND

In applying bubble models to characterize fluidized-bed hydrodynamics, one requires a predictive equation for the absolute upward velocity of the bubble phase in the bed. The commonly accepted equation is

$$u_B = u_o - u_{mf} + 0.711\,(gd_b)^{1/2} \tag{1}$$

which was presented originally by Nicklin (1962) for gas-liquid systems and later by Davidson and Harrison (1963) for gas-solid fluidized beds. It is observed from the above equation that the absolute bubble velocity, u_B, is the sum of the natural rising velocity, $0.711\,(gd_b)^{1/2}$, and the upward velocity of the dense-phase gas between bubbles, $(u_o - u_{mf})$.

The form of the natural rising velocity equation,

$$u_{br} = K_b(gd_b)^{1/2},$$

results from application of the steady-state mechanical energy balance to describe the rate of rise of a single spherical bubble in a low viscosity medium. The coefficient K_b is generally given the value 0.711 as obtained from experimental observations of Davies and Taylor (1950). However, a number of other investigators (Davidson et al., 1959; Donsi et al. 1972; Harrison and Leung, 1962;

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Park et al., 1969; Rippin, 1959; Toei et al., 1966) have assigned values to this coefficient ranging from 0.545 to 0.914. Nonetheless, Eq. 1 has been used consistently in all recent modeling efforts for describing bubble-phase velocity.

Davidson et al. (1977) have noted that the validity of Eq. 1 is doubtful. The natural rising bubble velocity term, $0.711\,(gd_b)^{1/2}$, can only be justified for the case of a single bubble and u_o approaching u_{mf} (from above). In addition, they note that the term, $(u_o - u_{mf})$, is questionable since it has been obtained by analogy with slug flow. Furthermore, they enumerate reasons for the actual bubble velocity being either smaller or larger than that predicted by Eq. 1. These are:

I. Actual bubble velocity less than predicted

- Interference between adjacent bubbles reducing velocity as for a slug rising more slowly in a smaller tube (Turner, 1966).
- It has been shown by Lockett et al. (1967) that percolation of gas can occur from bubble to bubble through the dense phase resulting in bubble flow being less than excess above minimum fluidization.
- Bubbles may be in regions of the bed where upward flow of gas is less than the cross-sectional average.

II. Actual bubble velocity greater than predicted

- Bubble velocity can be agumented by coalescence: as a trailing bubble rises into the wake of a bubble above, the following bubble tends to rise faster (Harrison and Leung, 1962).
- Bubbles may be located in the bed where the upward flow of gas is greater than the cross-sectional average

In this work, we will derive a bubble-phase velocity equation, similar to Eq. 1, but from concepts quite different from those applied by Davidson and Harrison. This improved equation can account for most of the factors cited above.

THEORETICAL DEVELOPMENT

Here, we consider a two-phase fluidized-bed system: a continuous dense phase consisting of solid particles surrounded by an interstitial gas and a dispersed bubble phase consisting of solids-free bubbles which develop and rise through the dense phase. Our primary concern is the unidimensional, upward, absolute velocity of the bubble phase.

Zuber and Findlay (1965) derived a general expression for the unidimensional velocity of the dispersed phase flowing through the continuous phase in any two-phase system. In terms of the fluidized-bed system, the weighted-mean, bubble-phase velocity is

$$\bar{u}_B = C_o \langle j \rangle + \bar{V}_{Bj} \tag{2}$$

where the mixture volumetric flux density is given by

$$\langle j \rangle = \frac{Q_B}{A_T} = \langle u_{Bo} \rangle \tag{3}$$

for a stagnant dense phase, and the distribution coefficient, C_o, is

$$C_o = \frac{\langle \delta_B j \rangle}{\langle \delta_B \rangle \langle j \rangle} = \frac{\dfrac{1}{A_T}\displaystyle\int_{A_T} \delta_B j\, dA_T}{\left[\dfrac{1}{A_T}\displaystyle\int_{A_T} \delta_B\, dA_T\right]\left[\dfrac{1}{A_T}\displaystyle\int_{A_T} j\, dA_T\right]} \tag{4}$$

This coefficient represents the effect of non-uniform bubble flow and volume fraction. The bubble-phase, weighted-mean, drift velocity term, $\bar{V}_{Bj}$, accounts for the effect of local relative velocity.

The bubble-phase drift velocity, V_{Bj}, is related to the local relative velocity between phases by

$$V_{Bj} = (1 - \delta_B) u_r \tag{5}$$

A relation for the bubble velocity relative to the dense-phase velocity, u_r, can be obtained from a mixture model developed by Ishii and Zuber (1979). This mixture model has been shown to describe bubble, droplet, and particulate flow for gas-liquid, liquid-liquid, and solid-liquid systems in the Stokes, Viscous, Newton's, Distorted Fluid Particle, and Churn-Turbulent flow regimes by a uniform method. Here, we consider application to a gas-solid, bubbling, fluidized-bed system characterized by a bubble phase and a dense phase. Drag similarity criteria and mixture properties are introduced. The analysis takes into account the effect of both the presence and motion of other bubbles. Characteristics of the bubbles are reflected in the single-bubble rise velocity.

Ishii and Zuber (1979) have developed from fundamental concepts a relative velocity relationship for the dispersed phase in a two-phase, multiparticle system:

$$u_r |u_r| = \frac{8}{3} = \frac{r_B}{C_D \rho_D} (\rho_D - \rho_B) g (1 - \delta_B) \tag{6}$$

A bubbling fluidized bed is characterized by turbulence and bubble coalescence. As the diameter of a bubble increases, the wake and bubble boundary layer can overlap due to the formation of a large wake region. A bubble can influence both the surrounding dense phase and other bubbles directly. Also, bubbles can become entrained in the wakes of other bubbles. This fluid behavior of large, deformed bubbles characterizes the Churn-Turbulent flow regime. These spherical-cap bubbles tend to manifest flat or inverted bases. Surface tension forces are unimportant, and the drag coefficient of a single, spherical-cap bubble in an infinite medium approaches a value of

$$C_{D\infty} = \frac{8}{3} \tag{7}$$

independent of bubble size.

In considering a multi-fluid-particle (here, the particles are bubbles), it is necessary to account for the effect of other particles (bubbles) on the flow field. Consequently, the drag coefficient, C_D, for a multiparticle system differs from C_D. In order to model the increased drag effect due to the presence of neighboring particles, Ishii and Zuber (1979) assumed a simple similarity criterion between single- and multi-particle systems based on an appropriate Reynolds number and derived the drag coefficient in a multiparticle (fluid or solid) system for the Churn-Turbulent flow regime. It is

$$C_D = \frac{8}{3}(1 - \delta_B)^2 \tag{8}$$

Hence, the relative velocity can be obtained from Eqs. 6 and 8 as

$$u_r = \left[\frac{\dfrac{8}{3} r_B (\rho_D - \rho_B) g}{\dfrac{8}{3}(1 - \delta_B)\rho_D}\right]^{1/2} = 0.71 \left[\frac{g d_b (\rho_D - \rho_B)}{\rho_D (1 - \delta_B)}\right]^{1/2} \tag{9}$$

The drift velocity of the bubble phase is now derived from Eqs. 5 and 9 to be

$$V_{Bj} = 0.71 \left[\frac{g d_b (\rho_D - \rho_B)(1 - \delta_B)}{\rho_D}\right]^{1/2} \tag{10}$$

It can also be shown that the weighted-mean drift velocity is given by

$$\bar{V}_{Bj} = 0.71 \left[\frac{g \bar{d}_b (\rho_D - \rho_B)(1 - \langle \delta_B \rangle)}{\rho_D}\right]^{1/2} \tag{11}$$

Finally, the improved bubble velocity equation is obtained from Eqs. 2, 3 and 10 as

$$\bar{u}_B = C_o \langle u_{Bo} \rangle + 0.71 \left[\frac{g \bar{d}_b (\rho_D - \rho_B)(1 - \langle \delta_B \rangle)}{\rho_D}\right]^{1/2} \tag{12}$$

in terms of average and weighted-mean values. A more detailed derivation of the above result is given elsewhere (Weimer, 1980).

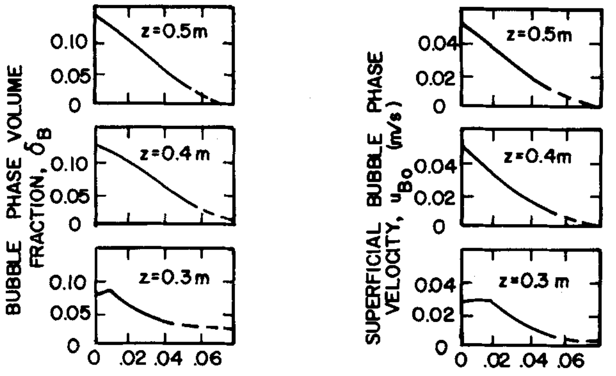

Figure 1. Variation of bubble-phase volume fraction and superficial bubble-phase velocity with axial and radial position in the fluidized bed as determined by Calderbank et al. (1976).

EXPERIMENTAL EVALUATION OF DISTRIBUTION COEFFICIENT

The distribution coefficient can be interpreted as a local value dependent on such factors as axial distance up the bed and gas velocity, or it can be viewed as an average value which is constant throughout the bulk of the fluidized bed and range of gas velocities considered—for a particular flow regime. Zuber and Findlay (1965) conclude that available experimental data show the distribution coefficient to remain constant for established flow profiles. However, at the time of their study, direct local measurements of bubble flow had not been made. We will show here that evaluations of such reported local measurements made from bubbling, gas-solid, fluidized beds indicate that the distribution coefficient varies with bed height and diameter, and inlet gas velocity.

In either case, the numerical value associated with the distribution coefficient indicates whether the bubble volume fraction is uniform or nonuniform over the bed cross-section. For a nonuniform coefficient, the value also indicates whether the bubbles are concentrated near the wall or near the centerline. In particular, if the bubbles are uniformly distributed across the bed cross-section,

$$\delta_{B_w} = \delta_{B_c} = \langle \delta_B \rangle \quad (13)$$

and

$$C_o = 1 \quad (14)$$

If the volume fraction of bubbles is greater toward the axial centerline, that is, if

$$\delta_{B_c} > \delta_{B_w} \quad (15)$$

then

$$C_o > 1 \quad (16)$$

and vice versa.

Zuber and Findlay (1965) have presented an extensive investigation of average volumetric concentration in two-phase flow systems. Their analysis of the distribution coefficient is presented in much detail, and this is recommended reading for those interested.

Local Values

The development of miniature capacitance probes (Werther and Molerus, 1973a; Werther, 1974a) has contributed to a better understanding of local bubble behavior in gas-solid fluidized beds. These devices have made possible the investigation of radial bubble volume fraction and flow distributions (Werther and Molerus, 1973a,b; Werther, 1974a,b,c, 1976; Burgess and Calderbank, 1975; Calderbank et al., 1976; Wittman et al., 1978). From the data now available, we can evaluate the distribution coefficient, C_o, directly via the definition given in Eq. 4. Each integral of Eq. 4, representing an average over the bed cross-section, may be determined from

$$\langle F \rangle = \frac{1}{A_T}\int_{A_T} F dA_T = \frac{2\pi}{\pi r_{\max}^2}\int_o^{r_{\max}} (rF)dr \quad (17)$$

for

$$F = \delta_B, j, \text{ and } \delta_B j.$$

TABLE 1. CALCULATIONS FOR DETERMINING DISTRIBUTION COEFFICIENT, C_o, FOR FLUIDIZED-BED SYSTEM OF CALDERBANK et al. (1976)

Bed Height, z(m)	0.3	0.4	0.5
$\langle j \rangle$ (m/s)	0.903	1.11	1.16
$\langle \delta_B \rangle$	0.0307	0.0354	0.0365
$\langle \delta_B j \rangle$ (m/s)	0.0376	0.0756	0.0830
$C_o = \frac{\langle \delta_B j \rangle}{\langle \delta_B \rangle \langle j \rangle}$	1.36	1.92	1.96

These integrals are evaluated numerically by quadrature using data taken from the above-referenced investigations.

Calderbank et al. (1976) have used miniature capacitance probes to make local measurements of bubble volume fraction, δ_B, absolute bubble rise velocity, u_B, and bubble diameter, d_b. Their bed was 0.155 m in diameter, and during operation the expanded height averaged 0.55 m. The fluidized solid had a minimum fluidization velocity of 0.011 m/s. The gas superficial velocity was maintained at 0.031 m/s. Local measurements were taken at elevations above the gas distributor of 0.3, 0.4 and 0.5 m, and along a radius of the bed to provide a complete traverse of the bubble dispersion. Their data are summarized in Figure 1. In viewing these profiles, it is apparent that bubbles are located closer to the axial centerline, and hence we should expect a distribution coefficient greater than unity. This is shown quantitatively in Table 1 where the three integrals of Eq. 4 have been evaluated numerically by quadrature. The distribution coefficients obtained, 1.36, 1.92 and 1.96, for increasing bed heights indicate that bubbles move toward the axial centerline as they travel up the bed.

Wittman et al. (1978) have made local measurements in a 0.19 m diameter fluidized bed. The collapsed bed height was 0.57 m, and the fluidized particles had a minimum fluidization velocity of 0.161 m/s. The inlet gas superficial velocity was varied in the range of two to four times that at minimum fluidization. Measurements were reported at axial distances of 0.055, 0.165, 0.372 and 0.565 m from the inlet gas distributor. Once again radial measurements were made, and the results are shown as radial profiles in Figure 2. It appears that bubbles are concentrated near the wall at heights of 0.055 and 0.165 m; whereas, at greater distances up from the gas distributor (0.372 and 0.565 m), bubbles approach the axial centerline. Calculations summarized in Table 2 yield values of the distribution coefficient from 1.07 to 2.56. The effect of height within the bed is clearly significant as C_o increases up the bed, that is, as bubbles tend toward the centerline. The effect of superficial gas velocity is not as clear. At a particular bed height, the distribution coefficient does vary with changes in gas velocity; however, the effect does not appear to be predictable.

It should be noted that some of our calculated values of the distribution coefficient are much higher than those reported by previous investigators. Zuber and Findlay (1965) found values of C_o ranging from 0.85 to 1.5 for gas-liquid systems. The value of 1.5 corresponds to a pronounced parabolic profile. As shown in Table 2, we have calculated directly values of the local distribution coefficient as high as 2.56. This indicates that bubbles in bubbling gas-solid fluidized beds do not travel up the bed in established flow profiles such as the familiar ones which develop for laminar flow in circular ducts.

Bubbles are known to travel up the bed following preferred paths (Geldart, 1970, 1971), and, in general, these paths approach the centerline.

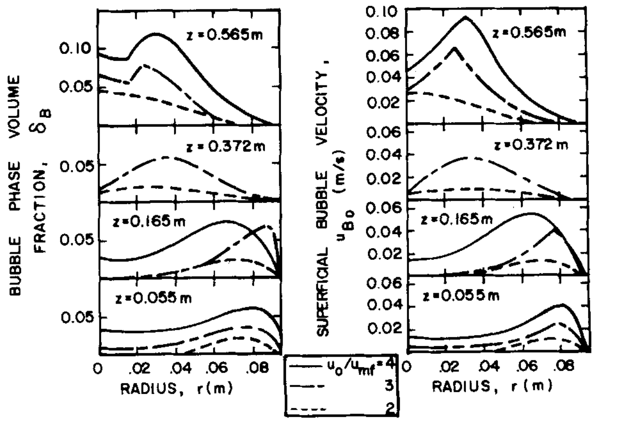

Figure 2. Variation of bubble-phase volume fraction and superficial bubble velocity with axial and radial position in the fluidized bed as determined by Wittman et al. (1978).

TABLE 2. CALCULATIONS FOR DETERMINING DISTRIBUTION COEFFICIENT, C_o, FOR FLUIDIZED-BED SYSTEM OF WITTMAN et al. (1978)

Bed Height, z(m)	0.055			0.165			0.372		0.565		
Ratio, u_o/u_{mf}	2	3	4	2	3	4	2	3	2	3	4
$\langle j \rangle$ m/s	0.496	1.14	2.32	0.738	2.03	3.87	0.589	1.65	0.723	1.60	3.08
$\langle \delta_B \rangle$	0.0137	0.0251	0.0441	0.0130	0.0291	0.0569	0.00990	0.0241	0.0110	0.0202	0.0404
$\langle \delta_{Bj} \rangle$ (m/s)	0.00730	0.0355	0.114	0.0118	0.0833	0.244	0.00850	0.0603	0.0188	0.825	0.233
$C_o = \dfrac{\langle \delta_{Bj} \rangle}{\langle \delta_B \rangle \langle j \rangle}$	1.07	1.24	1.11	1.23	1.41	1.11	1.46	1.52	2.36	2.56	1.87

If most bubbles are near the centerline and few are close to the wall, values of the distribution coefficient greater than 1.5 are conceivable. Also, as the rate of coalescence decreases up the bed, a point would be reached at which the distribution coefficient would level off. The calculations summarized in Table 1 corroborate this. At an axial distance of 0.3 m above the distributor, C_o takes on a value of 1.36. As bubbles continue to move toward the centerline, the coefficient increases to 1.92 at 0.4 m and 1.96 at 0.5 m.

An important question that we have yet to address is: Does the distribution coefficient remain constant for an established flow regime of a given system? As we mentioned earlier, Zuber and Findlay (1965) attest that for fully established and constant flow profiles, the value of C_o is fixed. From an analysis of the slug flow regime (Weimer, 1980), we would have to agree. However, for the churn-turbulent flow regime, we have shown from reported data that the distribution coefficient may take on values from 1.1 to 2.6. The flow regime is established as churn-turbulent; yet the bubble flow profile is not and local values of the distribution coefficient vary up the bed.

All calculations reported here have resulted in values of the distribution coefficient greater than unity. However, we have been limited in our analysis to axial elevations greater than 0.05 m. Results presented by Werther (1974a,b,c, 1976) and Werther and Molerus (1973a,b) show qualitatively that the bulk of bubbles close to the inlet gas distributor are adjacent to the wall. This implies a distribution coefficient less than unity. However, the above-cited work does not present enough information to allow direct calculation of the distribution coefficient in this lower region of the bed.

To summarize our findings concerning local values of the distribution coefficient as a function of bed height: close to the inlet gas distributor, bubbles are concentrated near the wall and C_o takes on values <1. As coalescence takes place and the larger bubbles tend toward the centerline, C_o approaches 1.5. Further up the bed and in the absence of slug flow, bubbles are not near the wall and larger values of C_o are attained. Furthermore, the value of C_o tends to level off at high elevations in the bed. For a transition to slug flow, the distribution coefficient will decrease as bubbles grow and spread out across the bed cross-section (Weimer, 1980).

Average Values

Thus far we have been concerned with local values of the distribution coefficient. These values have been determined from local bubble measurements taken at various axial and radial positions within the fluidized bed. However, data from local bubble measurements are scarce and the experimental technique for obtaining such data is quite involved. This, combined with the fact that no equations are available to predict a priori local values of C_o, leads us to consider values averaged throughout the bed and over the range of inlet gas velocities considered. Furthermore, average values of the distribution coefficient are readily determined from the more common measurements of visible bubble flow (i.e., superficial bubble phase velocity) and expanded bed height.

The weighted-mean absolute bubble phase velocity is given by Eq. 12. We now consider the absolute bubble phase velocity and the superficial bubble phase velocity to be mean values averaged throughout the bed volume for a particular inlet gas velocity. These mean values are denoted as $\tilde{u}_B$ and $\tilde{u}_{Bo}$, respectively. The distribution coefficient is interpreted to be constant throughout the bed volume and for all inlet gas velocities considered. This constant value, which represents an average over both the bed volume and all inlet gas velocities, is represented by $\tilde{C}_{o_m}$. Furthermore, Ishii and Zuber (1979) claim that the drift velocity is constant throughout the system (bed volume and inlet gas velocities) for the case of churn-turbulent bubble flow. Hence, it will be denoted as $\tilde{V}_{Bj_m}$. With these assumptions,

$$\tilde{u}_B = \tilde{C}_{o_m}\tilde{u}_{Bo} + \tilde{V}_{Bj_m} \tag{18}$$

Equation 18 is represented by a straight line on a plot of mean absolute bubble velocity, $\tilde{u}_B$, versus mean superficial bubble velocity, $\tilde{u}_{Bo}$. The slope of this line yields $\tilde{C}_{o_m}$ and the intercept is given by $\tilde{V}_{Bj_m}$.

Values of the superficial bubble phase velocity can be readily determined at various axial distances above the gas distributor from standard experiments. Averaging these values over the bed volume,

$$\tilde{u}_{Bo} = \frac{1}{L_f}\int_0^{L_f} \langle u_{Bo} \rangle dz \tag{19}$$

With a measurement of the expanded height of the fluidized bed, we have the bubble volume fraction averaged throughout the bed for a particular gas flow

$$\tilde{\delta}_B = 1 - \frac{L_{mf}}{L_f} = \frac{1}{L_f}\int_0^{L_f} \langle \delta_B \rangle dz \tag{20}$$

The absolute bubble velocity averaged throughout the bed is given by

$$\tilde{u}_B = \tilde{u}_{Bo}/\tilde{\delta}_B \tag{21}$$

This method of determining average values for the distribution coefficient, $\tilde{C}_{o_m}$, will be illustrated. It requires experimental data in which simultaneous measurements of visible bubble flow and expanded bed height are reported. This type of data is plentiful.

Geldart (1976) has considered the bed expansion and visible bubble flow for sandlike powders of various mean sizes and collapsed bed depths. The fluidized bed was 0.305 m in diameter and bed heights at minimum fluidization were varied in the range L_{mf} = 0.10 to 2.0 m. Here we consider the particular case of particles of mean size $\bar{d}_p$ = 128 μm and inlet superficial gas velocities exceeding minimum fluidization by 0.02, 0.04, and 0.06 m/s. The resulting mean superficial bubble velocity is plotted against the mean absolute bubble velocity in Figure 3. A least squares fit of these data yields an average distribution coefficient of $\tilde{C}_{o_m}$ = 1.39.

McGrath and Streatfield (1971) considered the fluidization of large particles in shallow gas-fluidized beds. The fluidized bed was rectangular, 0.305 m × 0.156 m, and 0.133 m deep at minimum fluidization. The fluidized particles had a mean diameter of $\bar{d}_p$ = 1,540 μm and a minimum fluidization velocity of u_{mf} = 0.55 m/s. A plot of mean superficial bubble velocity against mean absolute bubble velocity is shown in Figure 4. A least squares fit of these data yields an average distribution coefficient of $\tilde{C}_{o_m}$ = 1.29.

This method of determining a value for the distribution coefficient averaged over the bed volume and operating gas velocities has been successfully applied to gas-liquid (Zuber and Findlay, 1965) systems and gas-solid (Staub, 1980) transport lines. It provides a simple means of determining the average distribution coefficient which can be easily applied in modeling.

Values of the average distribution coefficient which we have determined

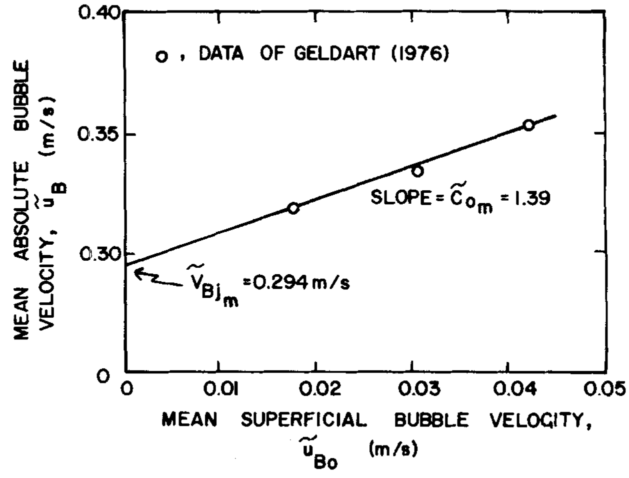

Figure 3. Mean absolute bubble velocity vs. mean superficial bubble velocity for experimental data of Geldart (1976).

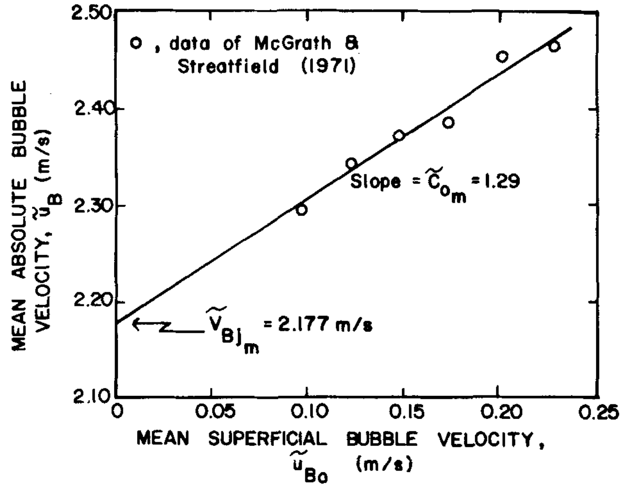

Figure 4. Mean absolute bubble velocity vs. mean superficial bubble velocity for experimental data of McGrath and Streatfield (1971).

here, $\tilde{C}_{o_m}$ = 1.29 and 1.39, appear to be reasonable in representing average bubble profiles throughout the bed. Both values are qualitatively representative of bubble profiles in shallow fluidized beds where it is improbable that bubbles could travel up the axial centerline in such a short distance. Hence, values of the local distribution coefficient would be expected to be less than $C_o = 1.50$.

COMPARISON OF IMPROVED BUBBLE VELOCITY EQUATION TO CONVENTIONALLY APPLIED BUBBLE VELOCITY EQUATION

It is appropriate to compare the improved bubble velocity equation, Eq. 12, to the conventionally applied bubble velocity equation, Eq. 1. It is apparent that the improved bubble velocity equation predicts an absolute bubble phase velocity less than that given in Eq. 1 when

1. Interference between adjacent bubbles reduces their velocities (i.e. as the averaged bubble phase volume fraction, $\langle \delta_B \rangle$, increases, the weighed-mean bubble drift velocity, $\bar{V}_{Bj}$, decreases)
2. The average superficial bubble velocity, $\langle u_{Bo} \rangle$, is less than the total inlet gas flow minus that required to incipiently fluidize the bed
3. Bubbles on the average are closer to the wall (i.e., $C_o < 1$)

The improved bubble phase velocity equation, Eq. 12, predicts a bubble-phase velocity which is greater than that given in Eq. 1 when bubbles on the average are closer to the bed axial centerline (i.e., $C_o > 1$), and this effect is greater than the slowing effects of bubble interference and of superficial bubble velocity being less than excess flow above minimum fluidization.

These differences result because the two equations have been derived from different concepts. As discussed earlier, the commonly accepted equation for absolute bubble velocity, Eq. 1, has been developed partially from analogy with slug flow and partially by applying the steady-state mechanical energy balance to describe the rate of rise of a single spherical bubble in a low viscosity medium (Davidson and Harrison, 1963). The bubble coefficient, K_b, was originally inferred from experimental results.

On the other hand, the improved bubble velocity equation, Eq. 12, has been developed by considering fundamental transport properties of the two phases, accounting for both the effect of non-uniform flow and bubble volume fraction as well as the effect of the local relative velocity between phases. Furthermore, consideration has been given to the influence of neighboring bubbles on the flow behavior of a characteristic bubble within the flux (i.e., considering multiparticle fluid properties).

A comparison of the two equations is best made by simplifying Eq. 12 to obtain the conventionally applied bubble velocity equation of Davidson and Harrison (1963), that is, Eq. 1. Assuming bubbles to be distributed evenly across the bed cross-section at any axial location, the distribution coefficient takes on the value

$$C_o = 1 \tag{22}$$

Assuming the relative velocity of the dispersed bubble phase to be unaffected by the presence of neighboring bubbles

$$u_{r_\infty} = u_r(1 - \delta_B)^{1/2} \quad \text{(Ishii and Zuber, 1979)} \tag{23}$$

From Eq. 5

$$u_r = \frac{V_{Bj}}{1 - \delta_B} = 0.71 \left[\frac{g d_b(\rho_D - \rho_B)}{\rho_D(1 - \delta_B)}\right]^{1/2} \tag{24}$$

Combining Eqs. 23 and 24

$$u_{r_\infty} = 0.71 \left[\frac{g d_b(\rho_D - \rho_B)}{\rho_D}\right]^{1/2} \tag{25}$$

With the dispersed bubble phase assumed to be described by a single bubble in an infinite medium unaffected by the presence of neighboring bubbles

$$V_{Bj} = u_{r_\infty} \tag{26}$$

Assuming all flow in excess of that required for minimum fluidization to contribute directly to the formation of bubbles

$$u_{Bo} = u_o - u_{mf} \tag{27}$$

Equations 2, 3, 22, 25, 26 and 27 can be combined to yield

$$\bar{u}_B = u_o - u_{mf} + 0.71 \left(\frac{g \bar{d}_b(\rho_D - \rho_B)}{\rho_D}\right)^{1/2} \tag{28}$$

in terms of weighted mean values. If,

$$\rho_D \gg \rho_B \tag{29}$$

then

$$(\rho_D - \rho_B)/\rho_D \approx 1 \tag{30}$$

and Eq. 28 is identical to Eq. 1 for the bubble phase.

ACKNOWLEDGMENTS

This work was made possible by a grant from the Department of Energy (Grant No. DE-FG22-80PC30245) and a fellowship from the then Department of Health, Education and Welfare Domestic Mining and Mineral Fellowship Program.

NOTATION

A_T	= cross-sectional area of the bed
C_o	= distribution coefficient defined in Eq. 4
$\tilde{C}_{o_m}$	= distribution coefficient averaged over bed volume and all operating conditions
C_D	= drag coefficient (multi-fluid-particle)
d_b	= bubble diameter (volume equivalent sphere)
d_p	= particle diameter
g	= gravitational constant
j	= velocity of the center of volume of dilute and dense phase, i.e., volumetric flux density of the mixture
K_b	= bubble coefficient used in Eq. 2
L_f	= expanded bed height
L_{mf}	= bed height at minimum fluidization
Q_B	= bubble volumetric flow
r	= radial distance
r_B	= bubble radius (equivalent spherical volume)
r_{max}	= radius of bed (tube)
u_o	= superficial gas velocity
u_{br}	= bubble natural rising velocity
u_B	= absolute velocity of bubbles, bubble phase
u_{Bo}	= superficial velocity of bubble phase
u_D	= absolute dense-phase (interstitial gas and particles) velocity
u_{mf}	= minimum fluidization velocity
u_r	= relative velocity = $u_B - u_D$
V_{Bj}	= drift velocity of bubble phase

Greek Letters

δ_B	= volume fraction bubbles
δ_{Bc}, δ_{Bw}	= bubble volume fraction—at axial centerline, at wall
ρ_B, ρ_D	= density of bubbles, dense phase

Superscripts

— = arithmetic mean value or a weighted mean value, i.e.,

$$\bar{F} = \frac{\langle \delta_B F \rangle}{\langle \delta_B \rangle}$$

~ = integrated average value over bed volume, i.e.,

$$\tilde{F} = \frac{1}{L_f} \int_0^{L_f} \langle F(z) \rangle dz$$

$\langle \ \rangle$ = average over cross-sectional area, i.e.,

$$\langle F \rangle = \frac{1}{A_T} \int_{A_T} F dA_T$$

Subscripts

∞ = property of a single bubble, unaffected by neighboring bubbles

c = value at cross-section centerline

m = average of value over all operating conditions

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Manuscript received December 1, 1981; revision received July 9, and accepted August 26, 1982.